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A SAFETY MEASURE OF STRUCTURAL CONTROL ACTUATORS

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1. INTRODUCTION

In recent years, there has been a growing interest in active control of acoustic radiation of vibrating structures such as an aircraft fuselage [1–8]. The structural control approach has been proven to be an effective solution for this problem. There are still some research issues that deserve further studies. One is related to sensing. The issue is how to develop practical acoustic sensor configurations that lead to global noise reduction in the enclosure. The other issue is related to the safety concern of structural controls. One potential problem with the structural control is that it may cause fatigue damage to structures [1, 6–8]. This issue is particularly sensitive to the aerospace industry. In this paper, a comparative study of the strain fields induced by several popular structural control actuators including distributed piezoelectric actuators is presented, and a measure of safety of structural control actuators developed. The measure is defined as a strain index [9].

In the following section, the numerical results of the comparative study are presented. In section 2, the motivation of and the need for a common index to evaluate the safety of different structural control actuators such as point forces, point moments, and distributed piezoelectric actuators are discussed. In section 3, a brief description of five structural control actuators is presented, and the numerical results of the strain index for each actuator.

2. THE STRAIN INDEX

The physical system used for the discussion is a uniform cylindrical shell with rigid endcaps as a simplified model of a section of fuselage. The shell is driven by a disturbance shaker that creates the sound inside. In order to reduce the noise in the shell, structural actuators have to produce secondary sound to cancel the primary one. While the noise is reduced, the structural vibration level may be increased by the control system [6–8]. One legitimate concern with the noise control by structural actuators is the potential fatigue damage to the structure. Although by combining structural vibration and acoustic pressure in the performance index, the noise and the structural vibration can be reduced at the same time, it is important to find out if there are localized high strains induced by the actuators that can potentially damage the structure.

In this paper, several popular structural control actuators are considered including a point force, a point moment, a rectangular piezoelectric patch, and the distributed piezoelectric acoustic and structural modal actuators as studied in reference [6]. In order to compare the strain fields induced by different actuators, there must be a common criterion for this purpose. The strain itself is not a proper criterion to evaluate the actuators. The reasons are as follows.

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Firstly, the input signals to the actuators are different. For the point force actuator, the input signal is force, and the unit is the Newton; for the point moment actuator, the input signal is moment, and the unit is the Newton-meter; for the piezoelectric actuators, the input signals are voltage, and the unit is the volt. The criterion must be invariant with respect to the input signals. Secondly, the spatial coupling to the shell changes with the actuator. The discrete actuators such as point forces and the point moments interact with the structure only at one point. The distributed actuators interact with the structure over a predetermined area. The shapes of the distributed actuators are also different from each other. Finally, the frequency responses for all the actuators are very different.

The authors propose a strain index for measuring the safety of structural control actuators. It is defined as

$$S_i = \frac{|\varepsilon_{max}(\omega)|^2}{U_{ave}(\omega)}\,, \tag{1}$$

where $\varepsilon_{max}(\omega)$ is the maximum principal strain over the entire surface of the shell, and $U_{ave}(\omega)$ is the spatially averaged structural potential energy. $\varepsilon_{\max}(\omega)$ can be obtained by numerically searching over the outer or inner surface of the shell. On the basis of equal average structural potential energy induced by the actuators, one would wish that the maximum strain of the shell is as small as possible. The strain field having a smaller strain index will be more uniform than that having a larger strain index. The strain index defined here is independent of the actuator input magnitude. This is true because both the numerator and denominator are proportional to the square of the actuator magnitude. The strain index is obviously frequency dependent and also a function of actuator location. In the following numerical results, the locations of all the actuators have been picked somewhat arbitrarily without a bias against or for any one actuator. Hence, the general trend of the strain index exhibited by all the actuators is indicative and useful.

3. NUMERICAL RESULTS AND REMARKS

The Donnell–Mushtari theory is used to model the cylindrical shell [10]. The model expansion method has been used to obtain the solutions to the equations of motion [6]. The shell is made of aluminium. It has a length 1·524 m, a radius 0·3048 m, and a thickness 2·38 mm. One focuses on five actuators in this study: a point force, a point moment, a rectangular patch actuator, a distributed acoustic modal piezoelectric actuator, and a distributed structural modal piezoelectric actuator. The general expressions of the forcing terms for each actuator are listed below. More detailed discussions on the modelling of these actuators can be found in reference [9].

The point force in the radial direction is

$$f_x(x,\theta,t)=0, \qquad f_\theta(x,\theta,t)=0, \qquad f_r(x,\theta,t)=f_0\,\delta(x-x_0)\,\delta(\theta-\theta_0)\,\mathrm{e}^{\mathrm{j}\omega t}. \tag{2}$$

The point moment in the circumferential direction is

$$f_x(x,\theta,t)=0, \qquad f_\theta(x,\theta,t)=0, \qquad f_r(x,\theta,t)=-m_0\,\delta(x-x_0)\,\delta'(\theta-\theta_0)\,\mathrm{e}^{\mathrm{j}\omega t}. \tag{3}$$

The force components of the piezoelectric actuators are

$$f_x(x,\theta,t)=-(h_{31}/\beta_{33})\,\partial s(x,\theta)/\partial x\;\,V_0\,\mathrm{e}^{\mathrm{j}\omega t}, \qquad f_\theta(x,\theta,t)=-(h_{32}/a\beta_{33})\,\partial s(x,\theta)/\partial\theta\;\,V_0\,\mathrm{e}^{\mathrm{j}\omega t},$$

$$f_r(x,\theta,t)=-\left\{\frac{h_{32}}{a\beta_{33}}\,s(x,\theta)-\frac{J_{pe}}{h_{pe}}\left[\frac{h_{32}}{a^2\beta_{33}}\,\frac{\partial^2 s(x,\theta)}{\partial\theta^2}+\frac{h_{31}}{\beta_{33}}\,\frac{\partial^2 s(x,\theta)}{\partial x^2}\right]\right\}V_0\,\mathrm{e}^{\mathrm{j}\omega t}, \tag{4}$$

where $s(x, \theta)$ is the shape function of the actuator. Other parameters can be found in references [6, 9]. For the rectangular patch, $s(x, \theta)$ is given by

$$s(x, \theta) = [\mathrm{H}(x - x_1) - \mathrm{H}(x - x_2)] \times [\mathrm{H}(\theta - \theta_1) - \mathrm{H}(\theta - \theta_2)], \tag{5}$$

where $\mathrm{H}(x)$ is the the Heaviside function. For the modal actuator, the shape function is given by

$$s(x, \theta) = \mathrm{H}(x - x_1) - \mathrm{H}(x - x_2), \qquad x_1 = x_0 - S(\theta)/2, \qquad x_2 = x_0 + S(\theta)/2, \tag{6}$$

where x_0 represents the center co-ordinate of the shape, $S(\theta)$ defines the width variation of the modal actuator in the circumferential direction, $S(\theta)$ can be written in terms of circumferential modes of the shell. The maximum width of the shape function is adjusted to be 0·03812 m. Note that the acoustic medium has the same set of circumferential modes as the shell. In the frequency range from 110–512 Hz, for example, the structural modes having a circumferential modal number less than four can couple well with the acoustic modes in that frequency range having the same circumferential mode number. For this reason, the distributed actuator that has a width variation $S(\theta)$ containing these lower order circumferential modes is called the acoustic modal actuator. The other actuator whose $S(\theta)$ function contains the circumferential modes of order higher than four is consequently called a structural modal actuator. This is because in this same frequency range, these structural modes cause the interior sound radiation only by the forced motion of the air, as opposed to inducing the acoustic resonance. For more discussions on the actuators, the reader is referred to [6, 9].

All the actuators are placed at 0·5 meters in the longitudinal direction. For the point force and the point moment, the circumferential positions are at 0·5236 rad. The piezoelectric rectangular patch of size 4 × 6 cm^2 is centered at (0·5 m, 0·5236 rad) with its edges parallel to the co-ordinates. The distributed piezoelectric acoustic and structural modal actuators are all around the shell. The frequency range used in simulations is from 100–512 Hz covering several acoustic and structural resonances of the system.

Figure 1 shows the main result of extensive simulations of all the five actuators. As can be seen from the figure, the distributed modal piezoelectric actuators produce lower strain index than the discrete actuators over the frequency range considered. Among the modal

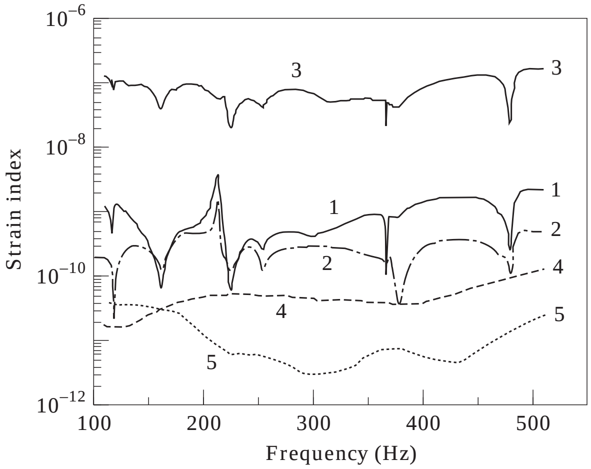

Figure 1. A comparison of strain index of the five control actuators: 1–1, the point force; 2–2, the point moment; 3–3, the rectangular patch piezoelectric actuator; 4–4 the acoustic modal piezoelectric actuator; 5–5, the structural modal piezoelectric actuator.

piezoelectric actuators, the acoustic modal actuator produces the higher strain index. This agrees with the authors' experimental observation. The result for the rectangular piezoelectric actuator is a surprise to the authors. One had anticipated it to produce a smaller strain index than the discrete actuators. The reason that it induces the highest strain index in the frequency range *may be* attributed to the spatial discontinuity of the actuator shape which leads to a line moment in the circumferential direction along two sides of its edge. The line moment is represented by a highly singular dipole function, and apparently produces far more localized strain than the discrete actuators which are singular only at one point.

4. CONCLUSIONS

A concept of strain index for comparing the safety of structural control actuators has been introduced, and a comparative study of strain fields on a uniform cylindrical shell induced by commonly used discrete and distributed control actuators presented. According to the simulation results, the strain index for the distributed modal piezoelectric actuators is much smaller than that for other actuators over a range of frequencies. This indicates that the distributed modal piezoelectric actuators induce more uniform strains than the discrete actuators as well as the actuators with spatial discontinuity such as rectangular patch. The results of the paper suggest that the distributed actuators with a smooth contour are safer than others.

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